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**DESIGN OF TOROIDAL TRANSFORMERS  
FOR MAXIMUM EFFICIENCY**

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16. Abstract <p>The design of the most efficient toroidal transformer that can be built given the frequency, volt-ampere rating, magnetic flux density, window fill factor, and materials is described here. With the above all held constant and only the dimensions of the magnetic core varied, the most efficient design occurs when the copper losses equal 60 percent of the iron losses. When this criterion is followed, efficiency is only slightly dependent on design frequency and fill factor. The ratios of inside diameter to outside diameter and height to build of the magnetic core that result in transformers of maximum efficiency are computed.</p>			
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# DESIGN OF TOROIDAL TRANSFORMERS FOR MAXIMUM EFFICIENCY

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## SUMMARY

In many electronic applications such factors as the efficiency of electrical power sources and the ratings of key components may determine the transformer current. However, no theorem exists for determining the design of the most efficient transformer when current is not a variable. This report provides a criterion for maximum efficiency for a toroidal transformer with a given current, frequency, volt-ampere rating, magnetic flux density, window fill factor, and materials.

The expressions derived are specifically for the toroidal transformer, both because this type has wide applicability and because its description may be reduced to dependence on a single dimension, height  $H$ , and two geometric factors, the ratio of inside to outside diameter  $Y$  and the ratio of height to build  $Z$ . However, appropriately restated, the basic results presented here probably hold for any type of transformer.

A continuum of wire and core sizes and shapes are permitted in the analysis. Two winding schemes are considered, the basic two winding transformer with equal current densities in both windings and the inverter transformer with a center tapped primary designed for twice the current density of the secondary.

As the dimensions of the transformer are varied, which in this case of fixed current correspondingly varies the current density in the windings, it is found that the most efficient design for a given  $Y$  and  $Z$  is reached when the conduction or copper losses equal 60 percent of the iron losses. It is shown that the efficiency can be maximized by varying  $Y$  and  $Z$  to the values  $Y_M$  and  $Z_M$ , which do not depend on any design parameter other than the fill factor. The values of  $Y_M$  and  $Z_M$  are determined for the two winding arrangements considered here, the basic two winding transformer and the inverter transformer with center-tapped primary.

Using this procedure it is shown that for given materials the efficiency of the transformer designed depends only slightly on the frequency.

A numerical example demonstrates that, at its maximum, transformer efficiency is only slightly dependent on fill factor.

## INTRODUCTION

Transformers are used in many applications where circuit efficiency is a primary concern. Previously, if a designer wished to maximize transformer efficiency there were no guidelines to follow in the case where current was not a design variable. Such cases arise, for example, when current must be limited by the ratings of other circuit components or when the power supply must operate at a particular current. The purpose of this report is to present a criterion for the maximization of toroidal transformer efficiency when current is not an appropriate variable.

The design equations of the toroidal transformer have been reduced to dependence on a single dimension, the height  $H$ , and two geometric parameters, the inside to outside diameter ratio  $Y$  and the height to build  $((D_O - D_I)/2)$  ratio  $Z$ . In this way, an analysis can be made of the dependence of efficiency of the toroidal transformer on core size and shape.

In a previous study (ref. 1) in which the design equations of the toroidal transformer were computerized, certain trends in efficiency were observed. However, these were somewhat obscured by the restriction of the program to core sizes that are available commercially as catalog items and to standard sizes of round wire. The present study allows a continuum of core sizes and shapes and permits an explicit expression for the core dimensions needed to produce the most efficient toroidal transformer possible at a fixed current, voltage, fill factor, magnetic flux density, core material, winding material, and operating temperature. A continuum of wire sizes is also assumed here, permitting the designer to choose whatever conducting material and shape the application dictates.

Two cases are considered: the simple transformer having an untapped primary and an untapped secondary and the basic parallel inverter transformer with a center-tapped primary and untapped secondary. The maximum efficiency criteria are derived, and some numerical examples are presented.

## SYMBOLS

$A_c$	effective cross sectional area of magnetic core, $m^2$
$A_{c, wi, 1}$	area of conducting portion of primary winding material, $m^2$
$A_{c, wi, 2}$	area of conducting portion of secondary winding material, $m^2$
$A_{T, wi, 1}$	area of primary winding material, $m^2$
$A_{T, wi, 2}$	area of secondary winding material, $m^2$
$A_W$	window area of core box, $m^2$

$a$	temperature coefficient of resistivity, $^{\circ}\text{C}^{-1}$
$B_M$	maximum magnetic flux density, T
$C_1$	function defined by eq. (35)
$C_2$	function defined by eq. (40)
$C_3$	function defined by eq. (36)
$C_4$	function defined by eq. (37)
$C_5$	function defined by eq. (38)
$D_I$	inside diameter of core iron, m
$D_{IT}$	inside diameter of core box, m
$D_O$	outside diameter of core iron, m
$D_{OT}$	outside diameter of core box, m
$D_1$	function defined by eq. (43)
$D_2$	function defined by eq. (49)
$D_3$	function defined by eq. (44)
$D_4$	function defined by eq. (46)
$D_5$	function defined by eq. (47)
$D_6$	function defined by eq. (45)
$e_1$	instantaneous voltage applied to primary, V
$F$	fill factor, dimensionless
$f$	frequency, Hz
$H$	height of core iron, m
$H_M$	height of core iron for maximum efficiency, m
$H_T$	height of core box, m
$I_1$	primary current, A
$I_2$	secondary current, A
$J$	current density, $\text{A}/\text{m}^2$
$J_M$	current density for maximum efficiency, $\text{A}/\text{m}^2$
$J_1$	primary current density of basic transformer, $\text{A}/\text{m}^2$
$k_1$	function defined by eq. (24)
$k_2$	function defined by eq. (25)

$L_1$	length of primary conductor, m
$L_2$	length of secondary conductor, m
$M$	mass of transformer, kg
$M_c$	mass of conducting material in transformer, kg
$M_i$	mass of transformer iron, kg
$N_p$	primary turns
$N_s$	secondary turns
$P_{cu}$	transformer conduction losses, W
$P_i$	transformer iron losses, W
$R_{eq}$	equivalent winding resistance reflected to primary, $\Omega$
$R_1$	primary resistance, $\Omega$
$R_2$	secondary resistance, $\Omega$
$T$	operating temperature, $^{\circ}\text{C}$
$T_0$	reference temperature in eq. (12), $^{\circ}\text{C}$
$t$	time, sec
$V_1$	primary voltage, V
$V_2$	secondary voltage, V
$W_f$	waveform factor, dimensionless
$W_i$	specific core loss, $\text{W}/(\text{kg})(\text{Hz})(\text{T})$
$W_{in}$	insulation increment, dimensionless
$Y$	diameter ratio, dimensionless
$Y_M$	diameter ratio for maximum efficiency, dimensionless
$Z$	height to build ratio, dimensionless
$Z_M$	height to build ratio for maximum efficiency, dimensionless
$\eta$	efficiency, percent
$\rho$	electrical resistivity of conduction material, $\Omega\text{m}$
$\rho_M$	density of magnetic material, $\text{kg}/\text{m}^3$
$\rho_{MC}$	density of conducting material, $\text{kg}/\text{m}^3$
$\rho_0$	resistivity at $T_0$ , $\Omega\text{m}$
$\varphi$	magnetic flux, We

## DERIVATION OF CONDITION OF MAXIMUM EFFICIENCY

The first case to be considered is the simple toroidal transformer having a single, untapped primary coil of  $N_P$  turns and a single, untapped secondary coil of  $N_S$  turns. Beginning with Faraday's Law,

$$e_1 = N_P \frac{d\varphi}{dt} \quad (1)$$

An expression for the effective voltage  $V_1$  induced in the primary coil may be derived as

$$V_1 = 4W_f f N_P B_m A_C \quad (2)$$

where  $W_f$  is the waveform factor (1.0 for a square wave, 1.11 for a sin wave),  $f$  is the frequency,  $B_m$  is the peak magnetic flux density, and  $A_C$  is the effective cross sectional area of the core.

A second relation is available that links the number of turns to the dimensions of the toroid,

$$N_P A_{T, wi, 1} = A_W \frac{F}{2} \quad (3)$$

where  $A_{T, wi, 1}$  is the total area of the primary wire,  $A_W$  is the effective area of the core window,  $F$  is the fraction of the core window to be filled with windings, and the secondary and primary occupy equal fractions of the window area.

The derivation that follows reduces the equations describing the toroidal transformer design to dependence on a single variable, the height  $H$  of the iron core. The first step in this development will be to describe the length of the transformer windings in order to compute their resistance.

In a previous publication (ref. 1) expressions were presented for the average length of conductor in a toroidal transformer. For the primary the conductor length  $L_1$  is

$$L_1 = N_P \left[ 2H_T + D_{OT} + D_{IT} \left( 1 - 2\sqrt{1 - \frac{F}{2}} \right) \right] \quad (4)$$

and for the secondary

$$L_2 = N_S \left[ 2H_T + D_{OT} + D_{IT} \left( 3 - 2\sqrt{1 - \frac{F}{2}} - 2\sqrt{1 - F} \right) \right] \quad (5)$$

where  $H_T$  is the height of the core box,  $D_{OT}$  is the outside diameter of the core box, and  $D_{IT}$  is the inside diameter of the core box.

Equations (4) and (5) are derived assuming that the secondary is wound on top of the primary as shown in figure 1. Furthermore, each winding completely fills its fraction of the window, leaving no voids, and is uniformly distributed around the toroid.

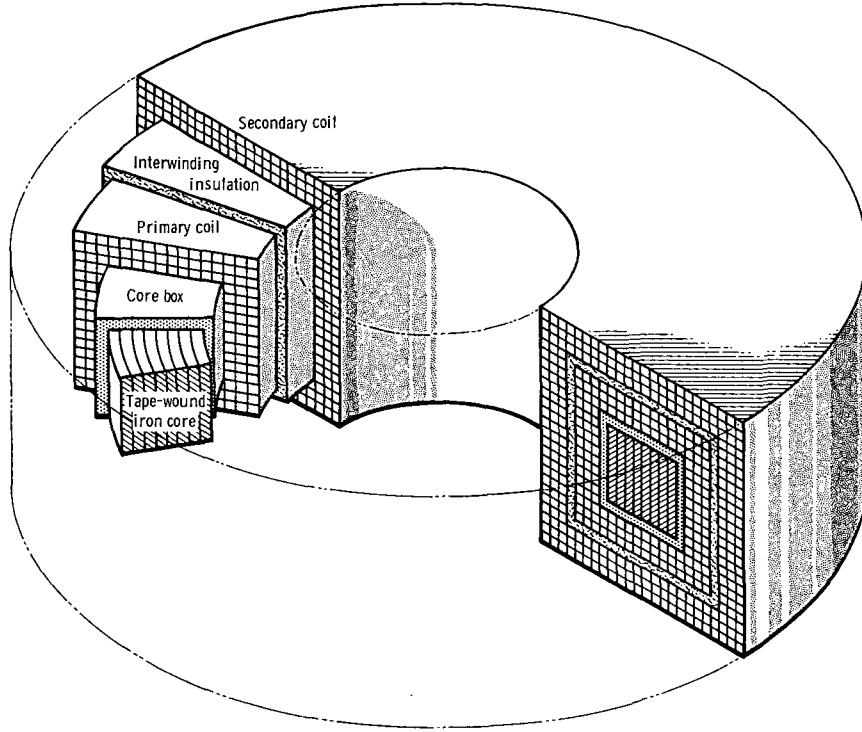


Figure 1. - Cutaway drawing of toroidal transformer with tape-wound core.

The resistance  $R_1$  of the primary coil and  $R_2$  of the secondary are then

$$R_1 = \frac{\rho L_1}{A_{c, wi, 1}} \quad (6)$$

$$R_2 = \frac{\rho L_2}{A_{c, wi, 2}} \quad (7)$$

where  $\rho$  is the effective resistivity of the conductor and  $A_{c, wi}$  is the area of the conducting portion of the winding material.

The area  $A_{c, wi, 1}$  is related to  $A_{T, wi, 1}$  by the relation

$$A_{T, wi, 1} = (1 + W_{in})A_{c, wi, 1} \quad (8)$$



where the increment  $W_{in}$  represents the increase in area of the coil due to the insulation of the wire and the introduction of insulating tape, if necessary.

When the corrections due to excitation current, leakage reactance, and winding resistance are neglected, the relation between voltage, current, and the number of turns in the transformer is simply

$$\frac{N_S}{N_P} = \frac{V_2}{V_1} = \frac{I_1}{I_2} \quad (9)$$

The equivalent transformer winding resistance  $R_{eq}$  reflected to the primary side is

$$R_{eq} = R_1 + R_2 \left( \frac{V_1}{V_2} \right)^2 \quad (10)$$

For the usual case in this simple transformer where the primary and secondary windings operate at the same current density, the equivalent resistance can be found after substitution of equations (4) to (7) and (9) into equation (10); it is

$$R_{eq} = \frac{\rho N_P}{A_{c, wi, 1}} \left[ 4H_T + 2D_{OT} + D_{IT} \left( 4 - 4\sqrt{1 - \frac{F}{2}} - 2\sqrt{1 - F} \right) \right] \quad (11)$$

When skin effect is eliminated through appropriate choice of conductor material, size, and shape,  $\rho$  is not a function of frequency and may be written as

$$\rho = \rho_0 \left[ 1 + a(T - T_0) \right] \quad (12)$$

For the case of copper conductors (ref. 2)  $\rho_0 = 1.73 \times 10^{-8}$  ohm-meter,  $a = 0.00393$   $^{\circ}\text{C}^{-1}$ , and  $T_0 = 20^{\circ}\text{C}$ .

The incremental dimension required for the core box is generally approximately 0.1 times the height  $H$  of the iron core. With the symbols  $Y$  used to denote the ratio of inside  $D_I$  to outside  $D_O$  diameters of the iron core and  $Z$  to signify the ratio of iron height  $H$  to iron thickness or build  $(D_O - D_I)/2$ , the dimensions of the core and the core box may all be expressed in terms of  $H$ ,  $Z$ , and  $Y$ .

$$H_T = 1.1H \quad (13)$$

$$D_O = \frac{2H}{Z(1 - Y)} \quad (14)$$

$$D_I = \frac{2YH}{Z(1 - Y)} \quad (15)$$

$$D_{OT} = H \left[ \frac{2}{Z(1 - Y)} + 0.1 \right] \quad (16)$$

$$D_{IT} = H \left[ \frac{2Y}{Z(1 - Y)} - 0.1 \right] \quad (17)$$

These equations may now be used to describe other core dimensions introduced in equations (2) and (3). Allowing 15 percent for insulation between layers of magnetic material, the effective core cross-sectional area is written

$$A_C = 0.85 \frac{H^2}{Z} \quad (18)$$

The window area is simply

$$A_w = \frac{\pi}{4} H^2 \left[ \frac{2Y}{Z(1 - Y)} - 0.1 \right]^2 \quad (19)$$

The expression for equivalent resistance can then be restated by substitution of equations (2), (3), (8), (13), and (16) to (19) into (11). This results in a relation for  $R_{eq}$  in terms of the basic design parameters for the toroidal transformer.

$$R_{eq} = \frac{\rho(1 + W_{in})V_1^2 Z^2 \left[ 2.1 + 0.2 \sqrt{1 - \frac{F}{2}} + 0.1 \sqrt{1 - F} + \frac{1}{Z(1 - Y)} \left( 2 + 4Y - 4Y \sqrt{1 - \frac{F}{2}} - 2Y \sqrt{1 - F} \right) \right]}{2.2698 B_M^2 f^2 W_f^2 F H^5 \left[ \frac{2Y}{Z(1 - Y)} - 0.1 \right]^2} \quad (20)$$

The total copper losses in the transformer  $P_{cu}$  are then simply

$$P_{cu} = I_1^2 R_{eq} \quad (21)$$

The iron losses  $P_i$  are approximated by taking the specific iron loss to vary linearly with frequency and flux density.

$$P_i = \pi \rho_M W_i f B_M H^3 \left( \frac{1+Y}{1-Y} \right) \left( \frac{1}{Z^2} \right) \quad (22)$$

where  $\rho_M$  is the density of the magnetic material and  $W_i$  is the specific core loss in watts per kilogram per hertz per tesla.

The efficiency  $\eta$  of the transformer, expressed in percent, is

$$\eta = \frac{100(V_1 I_1 - P_{cu} - P_i)}{V_1 I_1} = 100 - k_1 H^{-5} - k_2 H^3 \quad (23)$$

where

$$k_1 = \frac{44.06 \rho (1 + W_{in}) V_1 I_1 Z^2 \left[ 2.1 + 0.2 \sqrt{1 - \frac{F}{2}} + 0.1 \sqrt{1 - F} + \frac{1}{Z(1-Y)} \left( 2 + 4Y - 4Y \sqrt{1 - \frac{F}{2}} - 2Y \sqrt{1 - F} \right) \right]}{B_M^2 f^2 W_i^2 F \left[ \frac{2Y}{Z(1-Y)} - 0.1 \right]^2} \quad (24)$$

and

$$k_2 = \frac{100 \pi \rho_M W_i f B_M}{V_1 I_1} \left( \frac{1+Y}{1-Y} \right) \left( \frac{1}{Z^2} \right) \quad (25)$$

Since  $Y$  and  $F$  must be less than 1, the functions  $k_1$  and  $k_2$  are always real and positive.

When equation (23) is differentiated with respect to  $H$  the following relation is obtained

$$\frac{\partial \eta}{\partial H} = 5k_1 H^{-6} - 3k_2 H^2 \quad (26)$$

Since a second differentiation would yield a negative value, it can be stated that the efficiency of the transformer is maximized when

$$5k_1 H^{-6} - 3k_2 H^2 = 0 \quad (27)$$

or, multiplying through by  $H$  and referring to equation (23), when

$$P_{cu} = 0.6P_i \quad (28)$$

Thus as the core dimension  $H$  is varied, which is equivalent to varying the current density when  $I_1$ ,  $V_1$ ,  $B_M$ ,  $F$ ,  $f$ ,  $Y$ , and  $Z$  are all constant, the most efficient design is reached when copper losses are 60 percent of iron losses.

This result must not be confused with the conventional theorem of the most efficient operating point for a given transformer. That theorem is derived on the basis that only current is varied and results in the assertion that the most efficient operating point is reached when  $P_{cu} = P_i$ .

The expression derived here is of practical importance in any design where current is not a permitted variable and efficiency is critical.

Solving for  $H$  in equation (27) and designating this particular value of  $H$  as  $H_M$  results in

$$H_M = \left( \frac{5k_1}{3k_2} \right)^{1/8} \quad (29)$$

$$H_M = \left\{ \frac{0.734\rho(1 + W_{in})V_1^2I_1^2Z^4 \left[ 2.1 + 0.2\sqrt{1 - \frac{F}{2}} + 0.1\sqrt{1 - F} + \frac{1}{Z(1 - Y)} \left( 2 + 4Y - 4Y\sqrt{1 - \frac{F}{2}} - 2Y\sqrt{1 - F} \right) \right]}{\pi B_M^3 f^3 W_f^2 W_i F \rho_M \left[ \frac{2Y}{Z(1 - Y)} - 0.1 \right]^2 \left( \frac{1 + Y}{1 - Y} \right)} \right\}^{1/8} \quad (30)$$

The variation of the losses in the transformer designed to its most efficient point can be found by substituting  $H_M$  into equations (21) and (22):

$$P_{cu} \propto P_i \propto (B_M f)^{-1/8} \quad (31)$$

Therefore, when this criterion is applied in transformer design, the efficiency increases slowly as frequency is increased for constant  $B_M$ . However, this effect may be more than offset in practice by the nonlinearity of  $W_i$  at high frequencies.

The expression for efficiency can be rewritten by substituting equation (29) into equation (23) to obtain

$$\eta = 100 - 1.94k_1^{3/8}k_2^{5/8} \quad (32)$$

Inspecting equation (32) shows that the choice of transformer materials can be demonstrated to effect the most efficient designs with losses increasing as the  $5/8$  power of  $W_i$  and the  $3/8$  power of  $\rho$ .

The maximum efficiency that can be reached for a toroidal transformer designed with a fixed  $Y$ ,  $Z$ , and  $F$  is represented by equation (32). But the efficiency can be further improved by varying  $Y$  and  $Z$  for a particular value of  $F$ . The fill factor  $F$  will generally be controlled by the mechanics of transformer construction, so that it will be considered to be fixed. However, when  $F$  is available as a variable, it too can contribute to an ultimate enhancement of the efficiency.

The following expression is obtained by differentiating equation (32) with respect to  $Z$ , holding  $F$  and  $Y$  constant, and setting the result equal to zero:

$$3k_2 \frac{\partial k_1}{\partial Z} + 5k_1 \frac{\partial k_2}{\partial Z} = 0 \quad (33)$$

To implement this expression it is necessary to rewrite equation (24) in the form

$$k_1 = \frac{C_1 Z^2}{\left(\frac{C_3}{Z} - \frac{1}{10}\right)^2} \left(C_4 + \frac{C_5}{Z}\right) \quad (34)$$

where

$$C_1 = \frac{44.06\rho(1 + W_{in})V_1 I_1}{B_M^2 f^2 W_f^2 F} \quad (35)$$

$$C_3 = \frac{2Y}{1 - Y} \quad (36)$$

$$C_4 = 2.1 + 0.2\sqrt{1 - \frac{F}{2}} + 0.1\sqrt{1 - F} \quad (37)$$

and

$$C_5 = \frac{2 + 4Y - 4Y\sqrt{1 - \frac{F}{2}} - 2Y\sqrt{1 - F}}{1 - Y} \quad (38)$$

Equation (25) is rewritten

$$k_2 = \frac{C_2}{Z^2} \quad (39)$$

where

$$C_2 = \frac{100\pi\rho_M W_i f B_M}{V_1 I_1} \left( \frac{1 + Y}{1 - Y} \right) \quad (40)$$

After the indicated differentiations and substitutions are performed equation (33) becomes

$$0.4C_4 Z^2 + Z(2C_3 C_4 + 0.7C_5) - C_3 C_5 = 0 \quad (41)$$

Similarly, the variation of  $Y$  with  $Z$  and  $F$  fixed, leads to a maximization of efficiency. In this case equation (24) is rewritten as

$$k_1 = \frac{D_1 \left( D_3 + \frac{D_6 + D_4 Y}{1 - Y} \right)}{\left( \frac{D_5 Y}{1 - Y} - \frac{1}{10} \right)^2} \quad (42)$$

where

$$D_1 = \frac{44.06\rho(1 + W_{in})V_1 I_1 Z^2}{B_M^2 f^2 W_f^2 F} \quad (43)$$

$$D_3 = 2.1 + 0.2\sqrt{1 - \frac{F}{2}} + 0.1\sqrt{1 - F} = C_4 \quad (44)$$

$$D_6 = \frac{2}{Z} \quad (45)$$

$$D_4 = \frac{4 - 4\sqrt{1 - \frac{F}{2}} - 2\sqrt{1 - F}}{Z} \quad (46)$$

$$D_5 = \frac{2}{Z} \quad (47)$$

The expression for  $k_2$  is rewritten:

$$k_2 = D_2 \frac{(1 + Y)}{1 - Y} \quad (48)$$

where

$$D_2 = \frac{100\pi\rho_M W_1 f B_M}{Z^2 V_1 I_1} \quad (49)$$

Again, differentiating equation (32) with respect to  $Y$  and setting the result equal to zero result in

$$3k_2 \frac{\partial k_1}{\partial Y} + 5k_1 \frac{\partial k_2}{\partial Y} = 0 \quad (50)$$

The following is obtained by making the necessary substitutions and differentiations:

$$\begin{aligned} Y^2 [D_5(7D_4 - 4D_3 + 3D_6) + 1.3D_4 - D_3 + 0.3D_6] \\ + Y [D_5(7D_6 + 10D_3 - 3D_4) - D_4 + D_6 + 2D_3] - 6D_5(D_6 + D_3) \\ - 1.3D_6 - D_3 - 0.3D_4 = 0 \end{aligned} \quad (51)$$

The procedure and equations for maximizing efficiency described are applicable to any toroidal transformer. The particular expressions for  $k_1$  and  $k_2$  will depend on the winding configuration, however. For example, if the basic parallel inverter transformer is considered, that is, a transformer having a center tapped primary designed for twice the current density of the untapped secondary winding, the following adjustment must be made in  $k_1$  to account for the difference in primary resistance.

$$k_1 = \frac{44.06\rho(1 + W_{in})V_1I_1Z^2 \left[ 3.2 + 0.3\sqrt{1 - \frac{F}{2}} + 0.1\sqrt{1 - F} + \frac{1}{Z(1 - Y)} \left( 3 + 5Y - 6Y\sqrt{1 - \frac{F}{2}} - 2Y\sqrt{1 - F} \right) \right]}{f^2 B_M^2 F \left[ \frac{2Y}{Z(1 - Y)} - 0.1 \right]^2 W_f^2} \quad (52)$$

The maximization expressions (41) and (51) are unchanged, but new expressions for  $C_4$  and  $C_5$  must be used with equation (41) and for  $D_3$ ,  $D_4$ , and  $D_6$  in equation (51). These expressions are

$$C_4 = D_3 = 3.2 + 0.3\sqrt{1 - \frac{F}{2}} + 0.1\sqrt{1 - F} \quad (53)$$

$$C_5 = \frac{3 + 5Y - 6Y\sqrt{1 - \frac{F}{2}} - 2Y\sqrt{1 - F}}{1 - Y} \quad (54)$$

$$D_4 = \frac{5 - 6\sqrt{1 - \frac{F}{2}} - 2\sqrt{1 - F}}{Z} \quad (55)$$

$$D_6 = \frac{3}{Z} \quad (56)$$

When equations (41) and (51) are solved simultaneously, the values of  $Z$  and  $Y$  that maximize efficiency,  $Z_M$  and  $Y_M$ , are the result. That efficiency is maximized will be demonstrated by a numerical example in the next section. The values of  $Y_M$  and  $Z_M$  do not depend on the transformer materials, frequency, flux density, insulation thickness, signal waveform, temperature, or volt-ampere rating; they are functions solely of the type of winding and the fill factor. This is because  $C_1$ ,  $C_2$ ,  $D_1$ , and  $D_2$  drop out of



equations (41) and (51). Therefore, this geometrical relation need be solved only once for a particular type of toroidal transformer; the result will apply to all future designs.

The mass  $M$  of the copper and iron in the transformer can be computed by relying on the foregoing development. The mass of the copper  $M_c$  is simply

$$M_c = \rho_{MC}(L_1 A_{c, wi, 1} + L_2 A_{c, wi, 2}) \quad (57)$$

When equations (3) to (5) and (8), (9), (13), (16), and (17) are substituted into equation (57), the mass of the copper in both the simple and inverter transformer can be written

$$M_c = \frac{\pi \rho_{MC} F H^3}{8(1 + W_{in})} \left[ \frac{2Y}{Z(1 - Y)} - 0.1 \right]^2 \left[ 4.2 + 0.4 \sqrt{1 - \frac{F}{2}} + 0.2 \sqrt{1 - F} + \frac{4}{Z(1 - Y)} \left( 1 + 2Y - 2Y \sqrt{1 - \frac{F}{2}} - Y \sqrt{1 - F} \right) \right] \quad (58)$$

The mass of the iron  $M_i$  was already expressed in equation (22) as

$$M_i = \pi \rho_M H^3 \left( \frac{1 + Y}{1 - Y} \right) \left( \frac{1}{Z^2} \right) \quad (59)$$

It follows that  $M$  be the sum of equations (58) and (59):

$$M = M_c + M_i \quad (60)$$

The current density in the basic transformer is the same in both primary and secondary windings. For the primary of the basic transformer the current density  $J_1$  is written

$$J_1 = \frac{I_1}{A_{c, wi, 1}} \quad (61)$$

When equations (2), (3), and (8) are substituted into equation (61) the current density becomes

$$J_1 = \frac{V_1 I_1 (1 + W_{in})}{2W_f B_M F A_c A_W} \quad (62)$$

Further substitutions from equations (18) and (19) into equation (62) yield the expression for current density in the primary or secondary of the basic transformer or in the secondary of the inverter transformer. This general value of current density will be called simply  $J$  and expressed as

$$J = \frac{2.353 V_1 I_1 Z (1 + W_{in})}{\pi W_f B_M F H^4 \left[ \frac{2Y}{Z(1 - Y)} - 0.1 \right]^2} \quad (63)$$

The current density in the primary of the inverter transformer is twice  $J$ .

When the expression for  $H_M$  (given in eq. (30)), is substituted into equation (63) to obtain  $J_M$ , the current density at the most efficient point, it found that

$$J_M \propto f^{1/2} B_M^{1/2} \quad (64)$$

but is not a function of  $V_1$  or  $I_1$ .

## APPLICATIONS

That the solutions of equations (41) and (51) do indeed produce values of  $Y$  and  $Z$  which maximize efficiency is demonstrated by figure 2 where losses are plotted as a function of  $Z$  and  $Y$  for the inverter transformer. The magnitude described in figure 2 depends on the particular choice of materials, power rating, frequency, insulation, and magnetic flux density; the minimum, roughly in the neighborhood of  $Z = 1.5$ ,  $Y = 0.85$  for a fill factor of 0.5 (fig. 2(a)), would be the same for any transformer of this type. The minimum at a fill factor of 0.2 for the inverter transformer as seen in figure 2(b) is near  $Z = 1.0$ ,  $Y = 0.9$ . The data on Supermendur (ref. 3)(49 percent cobalt, 49 percent iron, 2 percent vanadium) was taken from Frost et al. (ref. 4).

A comparison of figures 2(a) and (b) indicates that the most efficient point tends to come at higher values of  $Y$  and lower values of  $Z$  as the fill factor is reduced. This trend is verified in figure 3 where  $Y_M$  and  $Z_M$ , found by iteratively solving equations (41) and (51) on a digital computer, are plotted as functions of fill factor in figure 3(a) for the basic transformer and in figure 3(b) for the inverter transformer. Also plotted on fig-

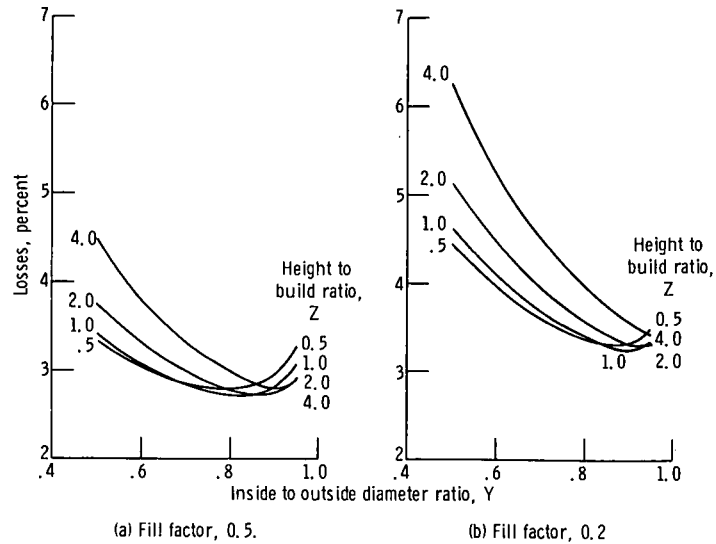


Figure 2. - Effect of core geometry on transformer losses. Inverter transformer; square-wave excited; Supermendur core; copper windings, temperature, 200° C; volt-ampereage, 2 kilovolt-amperes; frequency, 800 hertz; insulation increment, 0.125; maximum magnetic flux density, 1.8 tesla.

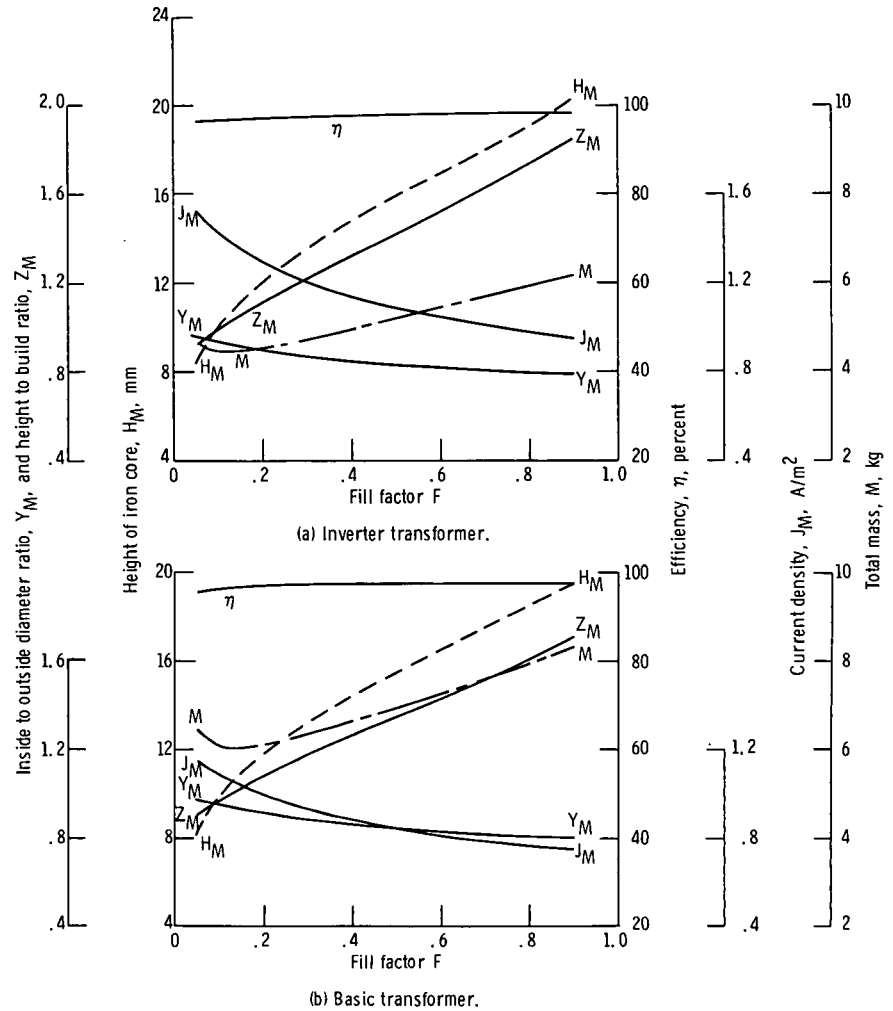


Figure 3. - Maximum efficiency conditions. Square-wave excitation, Supermendur core; copper winding; temperature, 200° C; volt-ampereage, 2 kilovolt-amperes; frequency, 800 hertz; insulation increment, 0.125; maximum magnetic flux density, 1.8 tesla.

ure 3 for purposes of illustration are the efficiency, transformer mass, current density, and height of the iron core. These four quantities depend on the specifications stated there, but  $Y_M$  and  $Z_M$  depend only on fill factor and transformer type.

The actual values of  $Y_M$  and  $Z_M$  approximated from figure 2 are found in figure 3(b). At a fill factor  $F$  of 0.2,  $Y_M = 0.91$  and  $Z_M = 1.07$ ; and at  $F = 0.5$ ,  $Y_M = 0.85$  and  $Z_M = 1.35$ .

## CONCLUSIONS

A new rule of thumb emerges from the results of this report. When current and voltage cannot be varied, but virtually all other design parameters are flexible, the most efficient design of a toroidal transformer will be that which produces copper losses equal to 60 percent of the iron losses.

A second result can be stated. For a given fill factor, the shape of the toroidal iron core can be specified to produce a transformer of the maximum efficiency possible for the materials and design used.

The first result may be generally true for any type of transformer whose design equations can be reduced to dependence on a single variable, such as the iron height used here, since the dimensional relation would probably be the same.

It should be noted that the most efficient design may not be the best design in every case. For example, by increasing current density somewhat from the value of  $J_M$  (current density for maximum efficiency), a smaller value of  $H$  (height of iron core) would be specified, and the transformer mass would be correspondingly lowered. The resulting reduction in efficiency might in many cases be more than compensated by the decrease in mass.

It should be particularly noted in figure 3 that by the correct choice of  $Y$  and  $Z$  efficiency is virtually unaffected by variations in fill factor. For the basic transformer, efficiency increases by only 0.6 percent for a variation in fill factor from 0.2 to 0.9. For the inverter transformer the change in efficiency is 0.7 percent over this range.

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112-27.

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